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A deformation of three dimensional BF theory

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Abstract: We consider a deformation of three dimensional BF theory by means of the antifield BRST formalism. Possible deformations for the action and the gauge symmetries are analyzed. We find a new class of gauge theories which include non-abelian BF theory, higher dimensional nonlinear gauge theory and topological membrane theory.

Keywords: Field Theories in Lower Dimensions, Topological Field Theories, Gauge Symmetry, BRST Symmetry.

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Contents

1. Introduction

The nonlinear gauge theory in two dimension are proposed in [1]. It is one of Schwarz type (or BF type) topological field theory [2] and has the gauge symmetry which generalize the usual nonabelian gauge symmetry. This theory was independently analyzed by Schaller and Strobl [3] by the Hamilton formalism.

This theory has some applications. One of them is two dimensional dilaton gravity [1, 4]. Recently, it is related to the string theory and a star product deformation theory. Cattaneo and Felder [5] have considered this theory on two dimensional disk. They obtained the path integral representation for the a star product on the Poisson manifold which was introduced by Kontsevich in [6]. The star product structure in the open string theory with non-zero background Neveu-Schwarz B-field appears essentially at the same mechanism [7].

Izawa [8] has recently analyzed the nonlinear gauge theory from the viewpoint of a deformation of the gauge symmetry [9]. He found that two dimensional nonlinear gauge theory is the unique consistent deformation of two dimensional abelian BF theory. He also found a higher dimensional nonlinear gauge theory.

In this paper we make a similar analysis in a higher dimension in three dimension and find deformations of the abelian BF theory,[1] which give new nontrivial extensions of the nonabelian gauge symmetry and nonlinear gauge symmetry. We find all

[1]Dayi have analyzed a deformation of the BF theories in a special case in [10].

deformations with a Lie algebra structure in three dimension. It includes a higher dimensional nonlinear gauge theory proposed in [8].

The key in nonlinear gauge theory is the nonlinear gauge symmetry $\delta_{\rm NL}$ on the fields:

$$\begin{aligned} \delta_{\rm NL}\phi_a &= W_{ba}\epsilon^b \\ \delta_{\rm NL}h^a &= d\epsilon^a + \frac{\partial W_{bc}}{\partial\phi_a}h^b\epsilon^c\,, \end{aligned} \tag{1.1}$$

where a, b, etc. are Lie algebra indices (or the target space indices). ϕ_a is a scalar field, h^a is a one-form gauge field and $W_{ab}(\phi) = -W_{ba}(\phi)$ is an arbitrary function of ϕ_a. $W_{ab}(\phi)$ must satisfy the following identities:

$$\frac{\partial W_{ab}}{\partial\phi_d}W_{cd} + \frac{\partial W_{bc}}{\partial\phi_d}W_{ad} + \frac{\partial W_{ca}}{\partial\phi_d}W_{bd} = 0\,, \tag{1.2}$$

in order for (1.1) to be a symmetry of the theory. This eq. (1.2) is just the Jacobi identity if the following commutation relation holds:

$$[\phi_a, \phi_b] = W_{ab}(\phi)\,. \tag{1.3}$$

The commutation relation on the left hand side is realized as the Poisson bracket of the coordinates ϕ_a and ϕ_b on the Poisson manifold [3]. That is, W_{ab} in (1.3) define the Poisson structure.

In two dimension, the action possessing the gauge symmetry (1.1) is uniquely given by

$$\begin{aligned} S &= \int \mathcal{L} \\ \mathcal{L} &= h^a d\phi_a + \frac{1}{2}W_{ab}h^a h^b\,. \end{aligned} \tag{1.4}$$

This paper is organized as follows. In section 2 we consider a deformations of three dimensional BF theory by means of antifield BRST formalism and construct a deformed new gauge theory. In section 3 we discuss the relations with the known theories.

2. A deformation of three dimensional BF theory

In three dimension, abelian BF theory has the following action:

$$S_0 = \int (A^a \wedge d\phi_a + B_a \wedge dh^a)\,, \tag{2.1}$$

where ϕ_a is a 0-form 'adjoint' scalar field, h^a and B_a are a 1-form and A^a is a 2-form gauge field. Indices a, b, c, etc. represent algebra indices. We can consider the more general term $A^a \wedge g(\phi)_a{}^b d\phi_b$ in the action, where $g(\phi)_a{}^b$ is a function of ϕ_a and similar one for the B, h term. Then if we redefine A^a as

$$A^{a\prime} = (g^{-1})^a{}_b(\phi) A^b \,, \tag{2.2}$$

we can obtain $A^{a\prime} \wedge d\phi_b$. It is similar for the B, h term.

It has the following abelian gauge symmetry:

$$\begin{aligned} &\delta_0 \phi_a = 0 \,, && \delta_0 h^a = d c_1{}^a \,, && \delta_0 c_1{}^a = 0 \,, \\ &\delta_0 B_a = d c_{2a} \,, && \delta_0 c_{2a} = 0 \,, && \\ &\delta_0 A^a = d c_3{}^a \,, && \delta_0 c_3{}^a = d v^a \,, && \delta_0 v^a = 0 \,, \end{aligned} \tag{2.3}$$

where $c_1{}^a$, c_{2a} are 0-form gauge parameters and $c_3{}^a$ is a 1-form gauge parameter. Since A^a is 2-form, we need a 'ghost for ghost' 0-form v^a.

In order to analyze the theory by the antifield BRST formalism, first we take $c_1{}^a$, c_{2a} and $c_3{}^a$ to be the Grassmann odd FP ghosts with ghost number one, and v^a to be a the Grassmann even ghost with ghost number two. Next we introduce antifields for all fields. Let Φ^* denote the antifields for the field Φ. The Batalin-Vilkovisky action which includes the antifields is given by

$$\begin{aligned} S_{BF} &= S_0 + S_1 \,, \\ S_1 &= \int (h^*_a \wedge d c_1{}^a + B^{*a} \wedge d c_{2a} + A^*_a \wedge d c_3{}^a + c^*_{3a} \wedge d v^a) \,. \end{aligned} \tag{2.4}$$

From this, the relations $\deg(\Phi^*) + \deg(\Phi) = 3$ and $\mathrm{gh}(\Phi^*) + \mathrm{gh}(\Phi) = -1$ are required, where we define $\deg(\Phi)$ and $\deg(\Phi^*)$ as the form degrees of the fields Φ and Φ^* and $\mathrm{gh}(\Phi)$ and $\mathrm{gh}(\Phi^*)$ as the ghost numbers. The BRST transformation can then be defined by

$$\delta_0 \Phi = (\Phi, S_{BF}) \,, \qquad \delta_0 \Phi^* = (\Phi^*, S_{BF}) \,, \tag{2.5}$$

where $(\cdot,\cdot)$ is the antibracket

$$(A, B) \equiv \frac{A \overleftarrow{\delta}}{\delta \Phi} \frac{\vec{\delta} B}{\delta \Phi^*} - \frac{A \overleftarrow{\delta}}{\delta \Phi^*} \frac{\vec{\delta} B}{\delta \Phi} \,, \tag{2.6}$$

for any field A and B. This transformation reproduces the gauge transformation (2.3) for the fields. Indeed the BRST transformation are obtained from (2.4) and (2.5) are given as follows:

$$\begin{aligned} &\delta_0 \phi_a = 0 \,, && \delta_0 h^a = d c_1{}^a \,, && \\ &\delta_0 B_a = d c_{2a} \,, && && \\ &\delta_0 A^a = d c_3{}^a \,, && \delta_0 c_3{}^a = d v^a \,, && \\ &\delta_0 \phi^*_a = d A^a \,, && \delta_0 h^*_a = - d B_a \,, && \delta_0 A^*_a = - d \phi_a \,, \\ &\delta_0 B^{*a} = - d h^a \,, && \delta_0 c^*_{1a} = - d h^*_a \,, && \delta_0 c^{*a}_2 = - d B^*_a \,, \\ &\delta_0 c^*_{3a} = d A^*_a \,, && \delta_0 v^{*a} = d c^*_{3a} \,, && \\ &\delta_0 \Phi = \delta_0 \Phi^* = 0 \,, \ \text{for otherwise} \,. && && \end{aligned} \tag{2.7}$$

In table 1, we show the form degrees and the ghost numbers for all the fields. The column and row correspond to the form degree and the ghost number, respectively.

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Let us consider a deformation to the action S_{BF} perturbatively,

$$
\begin{aligned}
S &= S_{BF} + gS_2 + g^2 S_3 + \cdots \\
&= S_0 + S_1 + gS_2 + g^2 S_3 + \cdots \,,
\end{aligned} \tag{2.8}
$$

where g is a coupling constant. The total BRST transformation is deformed to

$$
\delta\Phi = (\Phi, S)\,, \qquad \delta\Phi^* = (\Phi^*, S)\,. \tag{2.9}
$$

	0	1	2	3
-3				v^*_a
-2			c^*_{3a}	c^*_{1a}, c_2^{*a}
-1		A^*_a	B^{*a}, h^{*a}	ϕ^{*a}
0	ϕ_a	h^a, B_a	A^a	
1	${c_1}^a, c_{2a}$	${c_3}^a$		
2	v^a			

Table 1: Form degrees and ghost numbers of fields.

In order for the deformed BRST transformation δ to be nilpotent, the total action S has to satisfy the following master equation:

$$
(S, S) = 0\,. \tag{2.10}
$$

Substituting (2.8) to (2.10), we obtain

$$
(S, S) = (S_{BF}, S_{BF}) + 2g(S_2, S_{BF}) + g^2[(S_2, S_2) + 2(S_3, S_{BF})] + O(g^3) = 0\,. \tag{2.11}
$$

We solve this equation order by order. $\delta_0 S_{BF} = (S_{BF}, S_{BF}) = 0$ from the definition. At the first order of g in the eq. (2.11),

$$
\delta_0 S_2 = (S_2, S_{BF}) = 0\,, \tag{2.12}
$$

is required. S_2 is given by the lagrangian:

$$
S_2 = \int a_3\,, \tag{2.13}
$$

where a_3 must be a 3-form. A deformation of the lagrangian a_3 should obey the following descent equations:

$$
\begin{aligned}
\delta_0 a_3 + da_2 &= 0\,, \\
\delta_0 a_2 + da_1 &= 0\,, \\
\delta_0 a_1 + da_0 &= 0\,, \\
\delta_0 a_0 &= 0\,,
\end{aligned} \tag{2.14}
$$

where a_0 is a 0-form with the ghost number 3. where δ_0 cohomology on S is defined up to the terms proportional to the equations of motion. Because their terms can be eliminated by the field redefinitions and these are trivial deformations [9].

Since $\delta_0 a_0 = 0$, it should have the form

$$
\begin{aligned}
a_0 = &-f_{1ab}(\phi) v^a {c_1}^b - {f_{2a}}^b(\phi) v^a c_{2b} - \\
&-\frac{1}{6} f_{3abc}(\phi) {c_1}^a {c_1}^b {c_1}^c - \frac{1}{2} {f_{4ab}}^c(\phi) {c_1}^a {c_1}^b c_{2c} - \\
&-\frac{1}{2} {f_{5a}}^{bc}(\phi) {c_1}^a c_{2b} c_{2c} - \frac{1}{6} {f_6}^{abc}(\phi) c_{2a} c_{2b} c_{2c}\,,
\end{aligned} \tag{2.15}
$$

up to the BRST trivial terms. f_{1ab}, ${f_{2a}}^b$, f_{3abc}, ${f_{4ab}}^c$, ${f_{5a}}^{bc}$ and ${f_6}^{abc}$ are functions of ϕ_a to be fixed later. ${f_{4ab}}^c = -{f_{4ba}}^c$, ${f_{5a}}^{bc} = -{f_{5a}}^{cb}$, and f_{3abc} and f_6^{abc} are completely antisymmetric with respect to a, b, c. Here there are not the metric dependent terms such as $f_{ab}(\phi)\partial^\mu v^a \partial_\mu {c_1}^b$ in (2.15). Because these terms are proportional to the equations of motion and trivial in cohomology.

Solving (2.14) using (2.7), a_3 is obtained as follows:

$$
\begin{aligned}
a_3 = & -\frac{1}{6}\frac{\partial^3 f_{1ab}}{\partial\phi_c\partial\phi_d\partial\phi_e} A_e^* A_d^* A_c^* v^a {c_1}^b + \\
& + \frac{\partial^2 f_{1ab}}{\partial\phi_c\partial\phi_d}\left(-{c_3}_d^* A_c^* v^a c_1^b + \frac{1}{2} A_d^* A_c^* {c_3}^a {c_1}^b - \frac{1}{2} A_d^* A_c^* v^a h^b\right) + \\
& + \frac{\partial f_{1ab}}{\partial\phi_c}(v_c^* v^a {c_1}^b + {c_3}_c^* {c_3}^a {c_1}^b - {c_3}_c^* v^a h^b + A_c^* A^a {c_1}^b + A_c^* {c_3}^a h^b - A_c^* v^a B^{*b}) + \\
& + f_{1ab}(-\phi^{*a} {c_1}^b + A^a h^b + {c_3}^a B^{*b} + v^a c_2^{*b}) - \frac{1}{6}\frac{\partial^3 {f_{2a}}^b}{\partial\phi_c\partial\phi_d\partial\phi_e} A_e^* A_d^* A_c^* v^a c_{2b} + \\
& + \frac{\partial^2 {f_{2a}}^b}{\partial\phi_c\partial\phi_d}\left(-{c_3}_d^* A_c^* v^a c_{2b} + \frac{1}{2} A_d^* A_c^* {c_3}^a c_{2b} - \frac{1}{2} A_d^* A_c^* v^a B_b\right) + \\
& + \frac{\partial {f_{2a}}^b}{\partial\phi_c}(v_c^* v^a c_{2b} + {c_3}_c^* {c_3}^a c_{2b} - {c_3}_c^* v^a B_b + A_c^* A^a c_{2b} + A_c^* {c_3}^a B_b - A_c^* v^a h_b^*) + \\
& + {f_{2a}}^b(-\phi^{*a} c_{2b} + A^a B_b + {c_3}^a h_b^* + v^a c_{1b}^*) - \frac{1}{36}\frac{\partial^3 f_{3abc}}{\partial\phi_d\partial\phi_e\partial\phi_f} A_f^* A_e^* A_d^* {c_1}^a {c_1}^b {c_1}^c + \\
& + \frac{\partial^2 f_{3abc}}{\partial\phi_d\partial\phi_e}\left(-\frac{1}{6}{c_3}_e^* A_d^* {c_1}^a {c_1}^b {c_1}^c - \frac{1}{4} A_e^* A_d^* h^a {c_1}^b {c_1}^c\right) + \\
& + \frac{\partial f_{3abc}}{\partial\phi_d}\left(\frac{1}{6} v_d^* {c_1}^a {c_1}^b {c_1}^c - \frac{1}{2}{c_3}_d^* h^a {c_1}^b {c_1}^c - \frac{1}{2} A_d^* B^{*a} {c_1}^b {c_1}^c + \frac{1}{2} A_d^* h^a h^b {c_1}^c\right) + \\
& + f_{3abc}\left(\frac{1}{2} c_2^{*a} {c_1}^b {c_1}^c + B^{*a} h^b {c_1}^c + \frac{1}{6} h^a h^b h^c\right) - \\
& - \frac{1}{12}\frac{\partial^3 {f_{4ab}}^c}{\partial\phi_d\partial\phi_e\partial\phi_f} A_f^* A_e^* A_d^* {c_1}^a {c_1}^b c_{2c} + \\
& + \frac{\partial^2 {f_{4ab}}^c}{\partial\phi_d\partial\phi_e}\left(-\frac{1}{2}{c_3}_e^* A_d^* {c_1}^a {c_1}^b c_{2c} - \frac{1}{2} A_e^* A_d^* h^a {c_1}^b c_{2c} - \frac{1}{4} A_e^* A_d^* {c_1}^a {c_1}^b B_c\right) + \\
& + \frac{\partial {f_{4ab}}^c}{\partial\phi_d}\Big(\frac{1}{2} v_d^* {c_1}^a {c_1}^b c_{2c} - {c_3}_d^* h^a {c_1}^b c_{2c} - \frac{1}{2}{c_3}_d^* {c_1}^a {c_1}^b B_c - A_d^* B^{*a} {c_1}^b c_{2c} + \\
& \qquad + \frac{1}{2} A_d^* h^a h^b c_{2c} - A_d^* h^a {c_1}^b B_c - \frac{1}{2} A_d^* {c_1}^a {c_1}^b h_c^*\Big) + \\
& + {f_{4ab}}^c\left(c_2^{*a} {c_1}^b c_{2c} + B^{*a} h^b c_{2c} - B^{*a} {c_1}^b B_c + \frac{1}{2} h^a h^b B_c - h^a {c_1}^b h_c^* + \frac{1}{2} {c_1}^a {c_1}^b c_{1c}^*\right) - \\
& - \frac{1}{12}\frac{\partial^3 {f_{5a}}^{bc}}{\partial\phi_d\partial\phi_e\partial\phi_f} A_f^* A_e^* A_d^* {c_1}^a c_{2b} c_{2c} + \\
& + \frac{\partial^2 {f_{5a}}^{bc}}{\partial\phi_d\partial\phi_e}\left(-\frac{1}{2}{c_3}_e^* A_d^* {c_1}^a c_{2b} c_{2c} - \frac{1}{4} A_e^* A_d^* h^a c_{2b} c_{2c} + \frac{1}{2} A_e^* A_d^* {c_1}^a B_b c_{2c}\right) + \\
& + \frac{\partial {f_{5a}}^{bc}}{\partial\phi_d}\Big(\frac{1}{2} v_d^* {c_1}^a c_{2b} c_{2c} - \frac{1}{2}{c_3}_d^* h^a c_{2b} c_{2c} + {c_3}_d^* {c_1}^a B_b c_{2c} - \frac{1}{2} A_d^* B^{*a} c_{2b} c_{2c} +
\end{aligned}
$$

$$
+ A^*_d h^a B_b c_{2c} - A^*_d c_1{}^a h^*_b c_{2c} + \frac{1}{2} A^*_d c_1{}^a B_b B_c \Big) +
$$

$$
+ f_{5a}{}^{bc} \left(\frac{1}{2} c_2^{*a} c_{2b} c_{2c} + B^{*a} B_b c_{2c} - h^a h^*_b c_{2c} + \frac{1}{2} h^a B_b B_c - c_1{}^a c^*_{1b} c_{2c} - c_1{}^a h^*_b B_c \right) -
$$

$$
- \frac{1}{36} \frac{\partial^3 f_6{}^{abc}}{\partial\phi_d \partial\phi_e \partial\phi_f} A^*_f A^*_e A^*_d c_{2a} c_{2b} c_{2c} +
$$

$$
+ \frac{\partial^2 f_6{}^{abc}}{\partial\phi_d \partial\phi_e} \left(-\frac{1}{6} c^*_{3e} A^*_d c_{2a} c_{2b} c_{2c} - \frac{1}{4} A^*_e A^*_d B_a c_{2b} c_{2c} \right) +
$$

$$
+ \frac{\partial f_6{}^{abc}}{\partial\phi_d} \left(\frac{1}{6} v^*_d c_{2a} c_{2b} c_{2c} - \frac{1}{2} c^*_{3d} B_a c_{2b} c_{2c} - \frac{1}{2} A^*_d h^*_a c_{2b} c_{2c} + \frac{1}{2} A^*_d B_a B_b c_{2c} \right) +
$$

$$
+ f_6{}^{abc} \left(\frac{1}{2} c^*_{1a} c_{2b} c_{2c} + h^*_a B_b c_{2c} + \frac{1}{6} B_a B_b B_c \right), \tag{2.16}
$$

where the BRST trivial terms are dropped since their terms do not deform the BRST transformation. At the second order of g,

$$
(S_2, S_2) + 2(S_3, S_{BF}) = 0\,, \tag{2.17}
$$

is required. We cannot construct nontrivial S_3 which satisfies (2.17) from the integration of the local lagrangian. Therefore if we assume the locality of the action, we find $S_i = 0$ for $i \geq 3$. Then the condition (2.17) reduces to

$$
(S_2, S_2) = 0\,. \tag{2.18}
$$

This imposes the following conditions on the above equations f_i, $i = 1, \cdots, 6$:

$$
f_{1ae} f_{2b}{}^e + f_{2a}{}^e f_{1be} = 0\,, \tag{2.19}
$$

$$
\frac{\partial f_{1ac}}{\partial\phi_e} f_{1eb} + \frac{\partial f_{1ab}}{\partial\phi_e} f_{1ec} + f_{1ae} f_{4bc}{}^e + f_{2a}{}^e f_{3ebc} = 0\,, \tag{2.20}
$$

$$
- f_{1eb} \frac{\partial f_{2a}{}^c}{\partial\phi_e} + f_{2e}{}^c \frac{\partial f_{1ab}}{\partial\phi_e} + f_{1ae} f_{5b}{}^{ec} - f_{2a}{}^e f_{4eb}{}^c = 0\,, \tag{2.21}
$$

$$
f_{2e}{}^{[b} \frac{\partial f_{2a}{}^{c]}}{\partial\phi_e} + f_{1ae} f_6^{ebc} + f_{2a}{}^e f_{5e}{}^{bc} = 0\,, \tag{2.22}
$$

$$
f_{1e[b} \frac{\partial f_{4cd]}{}^a}{\partial\phi_e} - f_{2e}{}^a \frac{\partial f_{3bcd}}{\partial\phi_e} + f_{4e[b}{}^a f_{4cd]}{}^e + f_{3e[bc} f_{5d]}{}^{ae} = 0\,, \tag{2.23}
$$

$$
f_{1e[b} \frac{\partial f_{5c]}{}^{ad}}{\partial\phi_e} + f_{2e}{}^{[a} \frac{\partial f_{4bc}{}^{d]}}{\partial\phi_e} + f_{3ebc} f_6{}^{ead} + f_{4e[b}{}^{[d} f_{5c]}{}^{a]e} + f_{4bc}{}^e f_{5e}{}^{ad} = 0, \tag{2.24}
$$

$$
f_{1eb} \frac{\partial f_6{}^{acd}}{\partial\phi_e} - f_{2e}{}^{[a} \frac{\partial f_{5b}{}^{cd]}}{\partial\phi_e} + f_{4eb}{}^{[a} f_6{}^{cd]e} + f_{5e}{}^{[ac} f_{5b}{}^{d]e} = 0\,, \tag{2.25}
$$

$$
f_{2e}{}^{[a} \frac{\partial f_6{}^{bcd]}}{\partial\phi_e} - f_6{}^{e[ab} f_{5e}{}^{cd]} = 0\,, \tag{2.26}
$$

$$
f_{1e[a} \frac{\partial f_{3bcd]}}{\partial\phi_e} + f_{4[ab}{}^e f_{3cd]e} = 0\,, \tag{2.27}
$$

where $[\cdots]$ represents the antisymmetrization for the indices. For example, $\Phi_{[ab]} = \Phi_{ab} - \Phi_{ba}$.

Now we have obtained possible deformation of three dimensional BF theory. The deformed lagrangian is (2.16) and f_i's satisfy identities (2.19)– (2.27). The concrete transformation on each field is listed in the appendix. We set $g = 1$ in the later part of the paper.

If we set all antifields $\Phi^* = 0$ in (2.13), we obtain the usual classical action. we can write down it explicitly as

$$\begin{aligned} S &= \int \mathcal{L}\,, \\ \mathcal{L} &= A^a \wedge d\phi_a + B_a \wedge dh^a + f_{1ab} A^a h^b + f_{2a}{}^b A^a B_b + \\ &\quad + \frac{1}{6} f_{3abc} h^a h^b h^c + \frac{1}{2} f_{4ab}{}^c h^a h^b B_c + \frac{1}{2} f_{5a}{}^{bc} h^a B_b B_c + \frac{1}{6} f_6{}^{abc} B_a B_b B_c\,. \end{aligned} \tag{2.28}$$

There is a global symmetry $\phi \to \phi + \text{constant}$ in (2.1). However generally the deformed action (2.28) does not have such symmetry.

The equations of motion of this theory are

$$\begin{aligned} d\phi_a + f_{1ab} h^b + f_{2a}{}^b B_b &= 0\,, \\ dh^a + f_{2b}{}^a A^b + \frac{1}{2} f_{4bc}{}^a h^b h^c - f_{5b}{}^{ac} h^b B_c + \frac{1}{2} f_6{}^{abc} B_b B_c &= 0\,, \\ dB_a + f_{1ba} A^b + \frac{1}{2} f_{3abc} h^b h^c + f_{4ab}{}^c h^b B_c + \frac{1}{2} f_{5a}{}^{bc} B_b B_c &= 0\,, \\ -dA^a + \frac{\partial f_{1bc}}{\partial \phi_a} A^b h^c + \frac{\partial f_{2b}{}^c}{\partial \phi_a} A^b B_c + \frac{1}{6} \frac{\partial f_{3bcd}}{\partial \phi_a} h^b h^c h^d + & \\ + \frac{1}{2} \frac{\partial f_{4bc}{}^d}{\partial \phi_a} h^b h^c B_d + \frac{1}{2} \frac{\partial f_{5b}{}^{cd}}{\partial \phi_a} h^b B_c B_d + \frac{1}{6} \frac{\partial f_6{}^{bcd}}{\partial \phi_a} B_b B_c B_d &= 0\,. \end{aligned} \tag{2.29}$$

The BRST transformation δ on each field is calculated to be:

$$\begin{aligned} \delta\phi_a &= -f_{1ab} c_1{}^b - f_{2a}{}^b c_{2b}\,, \\ \delta h^a &= dc_1{}^a + f_{4bc}{}^a h^b c_1{}^c + f_{5b}{}^{ac} c_1{}^b B_c - \\ &\quad - f_{5b}{}^{ac} h^b c_{2c} + f_6{}^{abc} B_b c_{2c} - f_{2b}{}^a c_3{}^b\,, \\ \delta B_a &= f_{3abc} h^b c_1{}^c - f_{4ab}{}^c c_1{}^b B_c + \\ &\quad + dc_{2a} + f_{4ab}{}^c h^b c_{2c} + f_{5a}{}^{bc} B_b c_{2c} - f_{1ba} c_3{}^b\,, \\ \delta A^a &= \frac{\partial f_{1bc}}{\partial \phi_a} A^b c_1{}^c + \frac{1}{2} \frac{\partial f_{3bcd}}{\partial \phi_a} h^b h^c c_1{}^d - \frac{\partial f_{4bc}{}^d}{\partial \phi_a} h^b c_1{}^c B_d + \frac{1}{2} \frac{\partial f_{5b}{}^{cd}}{\partial \phi_a} c_1{}^b B_c B_d + \\ &\quad + \frac{\partial f_{2b}{}^c}{\partial \phi_a} A^b c_{2c} + \frac{1}{2} \frac{\partial f_{4bc}{}^d}{\partial \phi_a} h^b h^c c_{2d} + \frac{\partial f_{5b}{}^{cd}}{\partial \phi_a} h^b B_c c_{2d} + \frac{1}{2} \frac{\partial f_6{}^{bcd}}{\partial \phi_a} B_b B_c c_{2d} + \\ &\quad + dc_3{}^a + \frac{\partial f_{1bc}}{\partial \phi_a} c_3{}^b h^c + \frac{\partial f_{2b}{}^c}{\partial \phi_a} c_3{}^b B_c\,. \end{aligned} \tag{2.30}$$

We have obtained the above BRST transformation systematically by the antifield BRST formalism. Therefore (2.30) is the complete set of the BRST, that is, the gauge transformation. Generally, (2.30) is nilpotent only on shell. That is, the algebra of the symmetry is an open algebra.

We now consider the gauge symmetry algebra to understand the role of f_i's simply. We replace the ghost fields $c_1{}^a$, c_{2a} and $c_3{}^a$ to gauge parameters $\epsilon_1{}^a$, ϵ_{2a} and $\epsilon_3{}^a$. Then we obtain the usual gauge transformation. We decompose the gauge symmetry as

$$\delta = \epsilon_1{}^a\delta_{1a} + \epsilon_{2a}\delta_2{}^a + \epsilon_3{}^a\delta_{3a}\,. \tag{2.31}$$

Then the commutation relations of the algebra generators δ_{1a}, $\delta_2{}^a$ and δ_{3a} are derived from (2.30) and (2.29) as follows:

$$\begin{aligned}
[\delta_{1a},\delta_{1b}] &\approx f_{4ab}{}^c\delta_{1c} + f_{3abc}\delta_2{}^c - \left(\frac{\partial f_{3abd}}{\partial\phi_c}h^d + \frac{\partial f_{4ab}{}^d}{\partial\phi_c}B_d\right)\delta_{3c}\,,\\
[\delta_{1a},\delta_2{}^b] &\approx f_{5a}{}^{bc}\delta_{1c} - f_{4ac}{}^b\delta_2{}^c + \left(\frac{\partial f_{4ad}{}^b}{\partial\phi_c}h^d - \frac{\partial f_{5a}{}^{bd}}{\partial\phi_c}B_d\right)\delta_{3c}\,,\\
[\delta_2{}^a,\delta_2{}^b] &\approx f_6{}^{abc}\delta_{1c} + f_{5c}{}^{ab}\delta_2{}^c - \left(\frac{\partial f_{5d}{}^{ab}}{\partial\phi_c}h^d + \frac{\partial f_6^{abd}}{\partial\phi_c}B_d\right)\delta_{3c}\,,\\
[\delta_{1a},\delta_{3b}] &\approx \frac{\partial f_{1ab}}{\partial\phi_c}\delta_{3c}\,,\\
[\delta_2{}^a,\delta_{3b}] &\approx -\frac{\partial f_{2b}{}^a}{\partial\phi_c}\delta_{3c}\,,\\
[\delta_{3a},\delta_{3b}] &\approx 0\,.
\end{aligned} \tag{2.32}$$

Here $\approx$ means that the identities are satisfied only on shell, that is, up to the equations of motion. From (2.32), f_i's are thus seen to give the 'structure constants' in this algebra, although they actually depend on ϕ_a. Moreover we can see that the conditions (2.19) – (2.27) are nothing but the algebra closing conditions and the Jacobi identities on this Lie algebra.

3. Relations to the known 3D theories

3.1 Nonabelian gauge symmetry

First we consider a simple case. Let only $f_{4ab}{}^c$ be nonzero among six f_i's and be a constant. Then the conditions from (2.19) to (2.27) reduce to the following one:

$$f_{4e[d}{}^a f_{4bc]}{}^e = 0\,, \tag{3.1}$$

This is nothing but the identity for structure constants of a Lie algebra. Then we find that h^a has well known nonabelian gauge symmetry in (2.30), and the complete gauge transformation of the theory is obtained from (2.30) by setting $f_{4ab}{}^c$ a constant and the other f_i zero.

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3.2 Nonlinear gauge theory

Let W_{ab} be an arbitrary a, b antisymmetric function of ϕ_a. We take $f_{1ab} = W_{ab}$, $f_{4ab}{}^c = \partial W_{ab}/\partial\phi_c$ and other $f_i = 0$. Then (2.19) – (2.27) reduce to (1.2):

$$\frac{\partial W_{ab}}{\partial\phi_d} W_{cd} + \frac{\partial W_{bc}}{\partial\phi_d} W_{ad} + \frac{\partial W_{ca}}{\partial\phi_d} W_{bd} = 0\,, \tag{3.2}$$

and the action (2.28) coincides with the higher dimensional nonlinear gauge theory [8]. Moreover (2.30) give us the complete gauge transformation on this theory.

3.3 Chern-Simons-Witten gravity

Let us take $f_{4ab}{}^c = \epsilon_{ab}{}^c$, $f_6^{abc} = \Lambda\epsilon^{abc}$ and other $f_i = 0$, where ϵ^{abc} is three dimensional completely antisymmetric tensor and Λ is a constant. Then (2.19) – (2.27) are trivially satisfied. We rewrite $\omega^a \equiv h^a$, $e_a \equiv B_a$ for clarity. Then the action (2.28) becomes

$$\mathcal{L} = A^a \wedge d\phi_a + e_a \wedge d\omega^a + \frac{1}{2}\epsilon_{ab}{}^c\omega^a\omega^b e_c + \frac{\Lambda}{6}\epsilon^{abc} e_a e_b e_c\,. \tag{3.3}$$

A^a and ϕ_a completely decouple from the other fields. The remaining terms with e_a and ω^a are the Chern-Simons-Witten gravity action with a cosmological constant Λ [11]. We find that e_a is a dreibein and ω^a is a spin connection. The gauge transformation (2.30) reduces to

$$\begin{aligned}
\delta\phi_a &= 0\,,\\
\delta\omega^a &= dc_1{}^a + \epsilon_{bc}{}^a\omega^b c_1{}^c + \Lambda\epsilon^{abc} e_b c_{2c}\,,\\
\delta e_a &= -\epsilon_{ab}{}^c c_1{}^b e_c + dc_{2a} + \epsilon_{ab}{}^c\omega^b c_{2c}\,,\\
\delta A^a &= dc_3{}^a\,.
\end{aligned} \tag{3.4}$$

The transformation of ω^a and e_a are the gauge transformation founded in [11].

This theory also has the following the general coordinate and local Lorentz symmetry:

$$\begin{aligned}
\delta_G\phi_a &= -u^\lambda\partial_\lambda\phi_a\,,\\
\delta_G\omega^a &= -du^\lambda\omega_\lambda{}^a - u^\lambda\partial_\lambda\omega^a - ds^a - \epsilon_{bc}{}^a\omega^b s^c\,,\\
\delta_G e_a &= -du^\lambda e_\lambda{}^a - u^\lambda\partial_\lambda e_a - \epsilon_{ab}{}^c s^b e_c\,,\\
\delta_G A^a &= -\frac{1}{2}dx^\mu \wedge dx^\nu(\partial_\mu u^\lambda A_{\nu\lambda}{}^a + \partial_\nu u^\lambda A_{\mu\lambda}{}^a + u^\lambda\partial_\lambda A_{\mu\nu}{}^a),
\end{aligned} \tag{3.5}$$

where $e_a = dx^\mu e_{\mu a}$, $\omega^a = dx^\mu\omega_\mu{}^a$, $A^a = \frac{1}{2}dx^\mu \wedge dx^\nu A_{\mu\nu}{}^a$. u^μ and s^a are the general coordinate and local Lorentz transformation parameters. We note that (3.4) and (3.5) are redundant gauge transformations. If we turn on f_1 or f_2, we can modify the action (3.3) and couple A^a and ϕ_a with ω^a and e_a. However it is an unusual coupling and generally the general coordinate and local Lorentz symmetry of ω^a and e_a in (3.5) break down. Thus we cannot interpret ω^a and e_a as the spin connection and the dreibein in this case.

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3.4 Two-brane

Two dimensional nonlinear gauge theory (1.4) has been related to the string theory. If there is the inverse of W_{ab}, we can integrate h^a out, and then, we obtain

$$\mathcal{L} = -\frac{1}{2}(W^{-1})^{ab} d\phi_a d\phi_b \,. \tag{3.6}$$

This is the Neveu-Schwarz B-field part of the string world sheet action.

M-theory contains a two dimensional extended object. Its world volume action is three dimensional field theory. When there is a nonzero 3-from C^{abc}, its bosonic part is written as

$$S = -T\left(\int d^3x \sqrt{\det(-g_{\mu\nu})} + \frac{1}{3!}\int C^{abc} d\phi_a d\phi_b d\phi_c\right), \tag{3.7}$$

where $g_{\mu\nu} = \partial_\mu \phi_a \partial_\nu \phi_b \eta^{ab}$.

Now we consider 3-form field C^{abc} part in the world volume action, which is

$$S = \frac{T}{3!}\int C^{abc} d\phi_a d\phi_b d\phi_c \,, \tag{3.8}$$

where C^{abc} can generally depend on ϕ_a. This action can be rewritten to the first order form by introducing auxiliary fields η_a and A^a [12]:

$$S_{BSZ} = \int \left(A^a \wedge (d\phi_a - \eta_a) + \frac{T}{3!} C^{abc} \eta_a \eta_b \eta_c\right), \tag{3.9}$$

where η_a is a 1-from and A^a is 2-form. If we redefine η_a as

$$\eta_a \equiv -W_{ab} h^b \,, \tag{3.10}$$

then (3.9) becomes

$$S_{BSZ} = \int \left(A^a \wedge (d\phi_a + W_{ab} h^b) - \frac{T}{3!} C^{abc} W_{ad} W_{be} W_{cf} h^d h^e h^f\right). \tag{3.11}$$

Eq. (3.11) can be obtained by the following procedure from our action (2.28). We can introduce a coupling constant t by redefining B_a to tB_a. We take the limit $t \longrightarrow 0$ in the action (2.28). Then (2.28) coincide with (3.11) if we identify $f_{1ab} = W_{ab}$, $f_3^{abc} = TW_{ad}W_{be}W_{cf}C^{def}$. The theory at finite t has been unknown so far and is quite new one.

4. Conclusion and discussion

We considered all possible deformations of three dimensional BF theory by the antifield BRST formalism. It led us to a new gauge symmetry and a deformed new action, which includes any gauge symmetry deformation with a Lie algebra structure.

This gauge symmetry give an extension of the nonlinear gauge symmetry [1], and the action includes the three dimensional nonabelian BF theory, three dimensional Chern-Simons-Witten gravity theory and topological two-brane theory with nonzero 3-form C^{abc}.

Higher dimensional extension of our discussion is straightforward, it is interesting to investigate possible deformations of the BF theory in various dimension.

It will be possible to make the discussions analogous to Cattaneo and Felder [5], and then this theory may be related to a star product deformation theory or its extension. It will be also useful to examine possible relations with the M2 and M5-branes in M-theory.

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A. The total BRST transformations on all fields

The total BRST transformation on all fields without antifields are derived from (2.9) as follows:

$$
\begin{aligned}
\delta\phi_a &= -f_{1ab}{c_1}^b - {f_{2a}}^b c_{2b}\,, \\
\delta h^a &= dc_1 - \frac{\partial {f_{2b}}^a}{\partial\phi_c}A^*_c v^b - {f_{2b}}^a {c_3}^b + \frac{1}{2}\frac{\partial {f_{4bc}}^a}{\partial\phi_d}A^*_d {c_1}^b {c_1}^c + {f_{4bc}}^a h^b {c_1}^c - \\
&\quad -\frac{\partial {f_{5b}}^{ac}}{\partial\phi_d}A^*_d {c_1}^b c_{2c} - {f_{5b}}^{ac} h^b c_{2c} + {f_{5b}}^{ac}{c_1}^b B_c + \frac{1}{2}\frac{\partial {f_6}^{abc}}{\partial\phi_d}A^*_d c_{2b}c_{2c} + {f_6}^{abc}B_b c_{2c}\,, \\
\delta B_a &= dc_{2a} + \frac{\partial f_{1ba}}{\partial\phi_c}A^*_c v^b - f_{1ba}{c_3}^b + \frac{1}{2}\frac{\partial f_{3abc}}{\partial\phi_d}A^*_d {c_1}^b {c_1}^c + f_{3abc}h^b {c_1}^c + \\
&\quad +\frac{\partial {f_{4ab}}^c}{\partial\phi_d}A^*_d {c_1}^b c_{2c} + {f_{4ab}}^c h^b c_{2c} - {f_{4ab}}^c {c_1}^b B_c + \frac{1}{2}\frac{\partial {f_{5a}}^{bc}}{\partial\phi_d}A^*_d c_{2b}c_{2c} + {f_{5a}}^{bc}B_b c_{2c}\,, \\
\delta A^a &= d{c_3}^a - \frac{1}{2}\frac{\partial^3 f_{1de}}{\partial\phi_a\partial\phi_b\partial\phi_c}A^*_c A^*_b v^d {c_1}^e - \frac{\partial^2 f_{1cd}}{\partial\phi_a\partial\phi_b}c^*_{3b}v^c {c_1}^d + \frac{\partial^2 f_{1cd}}{\partial\phi_a\partial\phi_b}A^*_b {c_3}^c {c_1}^d - \\
&\quad -\frac{\partial^2 f_{1cd}}{\partial\phi_a\partial\phi_b}A^*_b v^c h^d + \frac{\partial f_{1bc}}{\partial\phi_a}A^b {c_1}^c + \frac{\partial f_{1bc}}{\partial\phi_a}{c_3}^b h^c - \frac{\partial f_{1bc}}{\partial\phi_a}v^b B^{*c} - \\
&\quad -\frac{1}{2}\frac{\partial^3 {f_{2d}}^e}{\partial\phi_a\partial-\phi_b\partial\phi_c}A^*_c A^*_b v^d c_{2e}\frac{\partial^2 - {f_{2c}}^d}{\partial\phi_a\partial\phi_b}c^*_{3b}v^c c_{2d} - + \frac{\partial^2 {f_{2c}}^d}{\partial\phi_a\partial\phi_b} - A^*_b {c_3}^c c_{2d} + \\
&\quad +\frac{\partial^2 {f_{2c}}^d}{\partial\phi_a\partial\phi_b}A^*_b v^c B_d + \frac{\partial {f_{2b}}^c}{\partial\phi_a}A^b c_{2c} + \frac{\partial {f_{2b}}^c}{\partial\phi_a}{c_3}^b B_c - \frac{\partial {f_{2b}}^c}{\partial\phi_a}v^b h^*_c - \\
&\quad -\frac{1}{12}\frac{\partial^3 f_{3def}}{\partial\phi_a\partial\phi_b\partial\phi_c}A^*_b A^*_c {c_1}^d {c_1}^e {c_1}^f - \frac{1}{6}\frac{\partial^2 f_{3cde}}{\partial\phi_a\partial\phi_b}c^*_{3b}{c_1}^c {c_1}^d {c_1}^e -
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}\frac{\partial^2 f_{3cde}}{\partial\phi_a\partial\phi_b}A^*_b h^c {c_1}^d {c_1}^e - \frac{1}{2}\frac{\partial f_{3bcd}}{\partial\phi_a}B^{*b}{c_1}^c{c_1}^d + \frac{1}{2}\frac{\partial f_{3bcd}}{\partial\phi_a}h^b h^c {c_1}^d - \\
&-\frac{1}{4}\frac{\partial^3 {f_{4de}}^f}{\partial\phi_a\partial\phi_b\partial\phi_c}A^*_b A^*_c {c_1}^d {c_1}^e c_{2f} - \frac{1}{2}\frac{\partial^2 {f_{4cd}}^e}{\partial\phi_a\partial\phi_b}c^*_{3b}{c_1}^c{c_1}^d c_{2e} - \\
&-\frac{\partial^2 {f_{4cd}}^e}{\partial\phi_a\partial\phi_b}A^*_b h^c {c_1}^d c_{2e} - \frac{1}{2}\frac{\partial^2 {f_{4cd}}^e}{\partial\phi_a\partial\phi_b}A^*_b {c_1}^c {c_1}^d B_e - \\
&-\frac{\partial {f_{4bc}}^d}{\partial\phi_a}B^{*b}{c_1}^c c_{2d} + \frac{1}{2}\frac{\partial {f_{4bc}}^d}{\partial\phi_a}h^b h^c c_{2d} - \\
&-\frac{\partial {f_{4bc}}^d}{\partial\phi_a}h^b {c_1}^c B_d - \frac{1}{2}\frac{\partial {f_{4bc}}^d}{\partial\phi_a}{c_1}^b {c_1}^c h^*_d - \\
&-\frac{1}{4}\frac{\partial^3 {f_{5d}}^{ef}}{\partial\phi_a\partial\phi_b\partial\phi_c}A^*_b A^*_c {c_1}^d c_{2e} c_{2f} - \frac{1}{2}\frac{\partial^2 {f_{5c}}^{de}}{\partial\phi_a\partial\phi_b}c^*_{3b}{c_1}^c c_{2d} c_{2e} - \\
&-\frac{1}{2}\frac{\partial^2 {f_{5c}}^{de}}{\partial\phi_a\partial\phi_b}A^*_b h^c c_{2d} c_{2e} + \frac{\partial^2 {f_{5c}}^{de}}{\partial\phi_a\partial\phi_b}A^*_b {c_1}^c B_d c_{2e} - \\
&-\frac{1}{2}\frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}B^{*b}c_{2c}c_{2d} + \frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}h^b B_c c_{2d} - \frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}{c_1}^b h^*_c c_{2d} + \\
&+\frac{1}{2}\frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}{c_1}^b B_c B_d - \frac{1}{12}\frac{\partial^3 {f_6}^{def}}{\partial\phi_a\partial\phi_b\partial\phi_c}A^*_b A^*_c c_{2d}c_{2e}c_{2f} - \\
&-\frac{1}{6}\frac{\partial^2 {f_6}^{cde}}{\partial\phi_a\partial\phi_b}c^*_{3b}c_{2c}c_{2d}c_{2e} - \frac{1}{2}\frac{\partial^2 {f_6}^{cde}}{\partial\phi_a\partial\phi_b}A^*_b B_c c_{2d}c_{2e} - \frac{1}{2}\frac{\partial {f_6}^{bcd}}{\partial\phi_a}h^*_b c_{2c}c_{2d} + \\
&+\frac{1}{2}\frac{\partial {f_6}^{bcd}}{\partial\phi_a}B_b B_c c_{2d}\,, \\
\delta {c_1}^a =\; & {f_{2b}}^a v^b + \frac{1}{2}{f_{4bc}}^a {c_1}^b {c_1}^c - {f_{5b}}^{ac}{c_1}^b c_{2c} + \frac{1}{2}{f_6}^{abc}c_{2b}c_{2c}\,, \\
\delta c_{2a} =\; & f_{1ba}v^b + \frac{1}{2}f_{3abc}{c_1}^b{c_1}^c + {f_{4ab}}^c {c_1}^b c_{2c} + \frac{1}{2}{f_{5a}}^{bc}c_{2b}c_{2c}\,, \\
\delta {c_3}^a =\; & dv^a - \frac{\partial^2 f_{1cd}}{\partial\phi_a\partial\phi_b}A^*_b v^c {c_1}^d + \frac{\partial f_{1bc}}{\partial\phi_a}{c_3}^b {c_1}^c - \frac{\partial f_{1bc}}{\partial\phi_a}v^b h^c - \\
&-\frac{\partial^2 {f_{2c}}^d}{\partial\phi_a\partial\phi_b}A^*_b v^c c_{2d} + \frac{\partial {f_{2b}}^c}{\partial\phi_a}{c_3}^b c_{2c} - \frac{\partial {f_{2b}}^c}{\partial\phi_a}v^b B_c - \\
&-\frac{1}{6}\frac{\partial^2 f_{3cde}}{\partial\phi_a\partial\phi_b}A^*_b {c_1}^c {c_1}^d {c_1}^e - \frac{1}{2}\frac{\partial f_{3bcd}}{\partial\phi_a}h^b {c_1}^c {c_1}^d - \\
&-\frac{1}{2}\frac{\partial^2 {f_{4cd}}^e}{\partial\phi_a\partial\phi_b}A^*_b {c_1}^c {c_1}^d c_{2e} - \frac{\partial {f_{4bc}}^d}{\partial\phi_a}h^b {c_1}^c c_{2d} - \frac{1}{2}\frac{\partial {f_{4bc}}^d}{\partial\phi_a}{c_1}^b {c_1}^c B_d - \\
&-\frac{1}{2}\frac{\partial^2 {f_{5c}}^{de}}{\partial\phi_a\partial\phi_b}A^*_b {c_1}^c c_{2d} c_{2e} - \frac{1}{2}\frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}h^b c_{2c}c_{2d} + \frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}{c_1}^b B_c B_d - \\
&-\frac{1}{6}\frac{\partial^2 {f_6}^{cde}}{\partial\phi_a\partial\phi_b}A^*_b c_{2c}c_{2d}c_{2e} - \frac{1}{2}\frac{\partial {f_6}^{bcd}}{\partial\phi_a}B_b c_{2c}c_{2d}\,, \\
\delta v^a =\; & \frac{\partial f_{1bc}}{\partial\phi_a}v^b {c_1}^c + \frac{\partial {f_{2b}}^c}{\partial\phi_a}v^b c_{2c} + \frac{1}{6}\frac{\partial f_{3bcd}}{\partial\phi_a}{c_1}^b {c_1}^c {c_1}^d + \frac{1}{2}\frac{\partial {f_{4bc}}^d}{\partial\phi_a}{c_1}^b {c_1}^c c_{2d} + \\
&+\frac{1}{2}\frac{\partial {f_{5b}}^{cd}}{\partial\phi_a}{c_1}^b c_{2c}c_{2d} + \frac{1}{6}\frac{\partial {f_6}^{bcd}}{\partial\phi_a}c_{2b}c_{2c}c_{2d}\,.
\end{aligned}
\tag{A.1}
$$

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